Killara HIGH SCHOOL

Student Number

2015

TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

Extension 1 Mathematics

21st July 2015

General Instructions

- Reading time – 5 minutes
- Working tine - 2 hours
- Write using blue or black pen
  Black pen is preferred
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11-14 show relevant mathematical reasoning and/or calculations
- Start a new booklet for each question

Total Marks – 70

Section I - Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II - Pages 6 - 11

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Question	Marks
1 - 10	/10
11	/15
12	/15
13	/15
14	/15
Total	/70

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

This assessment task constitutes 40% *of the Higher School Certificate Course Assessment*

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1 – 10 (Detach from paper)

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1 If $(2k+1)x^2-(k+2)x+1$ is a perfect square then the value of k is:

(A) 0

(B) 4

(C) 0 and 4

(D) –2

2 The point P divides the interval from A (-2, 2) to B (8, -3) internally in the ratio 3: 5.

What is the x –coordinate of P?

(A) $\frac{1}{2}$

(B) $\frac{7}{4}$

(C) $\frac{17}{4}$

(D) 3

3 The polynomial equation $3x^3-2x^2+3x-4=0$ has roots α, β *and* γ. The exact value of $\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\alpha\gamma}$ is:

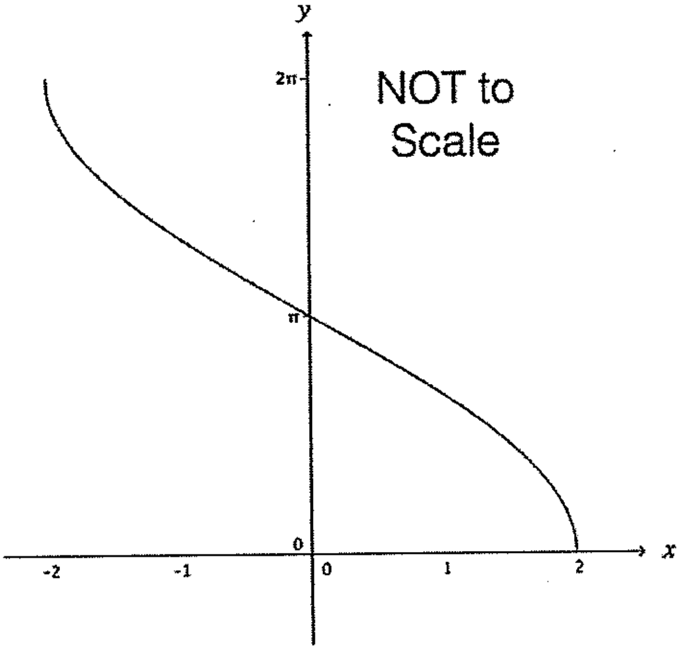
(A) 2

(B) –2

(C) $\frac{1}{2}$

(D) $-\frac{1}{2}$

4 What is the equation of the graph below:



(A) $y=2\sin^{-1}(2x)$

(B) $y=2\sin^{-1}\left(\dfrac{x}{2}\right)$

(C) $y=2\cos^{-1}(2x)$

(D) $y=2\cos^{-1}\left(\dfrac{x}{2}\right)$

5 In how many ways can 6 different keys be arranged on a circular key ring?

(A) 720

(B) 120

(C) 60

(D) 6

6 The sum of the infinite geometric series $1+2^n+2^{2n}+\dots$ is 2. The value of *n is:*

(A) 3

(B) $\frac{1}{2}$

(C) -1

(D) 2

7 Which expression is equivalent to $\int \cos^2 4x\,dx$?

(A) $\frac{1}{2}\left(\frac{1}{8}\sin 8x - x\right)+C$

(B) $\frac{1}{2}\left(\frac{1}{8}\sin 8x + x\right)+C$

(C) $\frac{1}{2}\left(\frac{1}{4}\sin 4x - x\right)+C$

(D) $\frac{1}{2}\left(\frac{1}{4}\sin 4x + x\right)+C$

8 When a polynomial is divided by $(x-2)(x+4)$ the remainder is $2x-5$.

What is the remainder when it is divided by $x-2$?

(A) - 5

(B) - 1

(C) 1

(D) 5

9 What is the derivative of $\sin^{-1}(2x)$

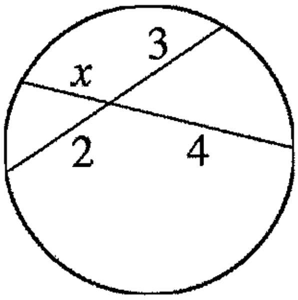
(A) $\dfrac{1}{2\sqrt{1-4x^2}}$

(B) $\dfrac{-1}{2\sqrt{1-4x^2}}$

(C) $\dfrac{2}{\sqrt{1-4x^2}}$

(D) $\dfrac{-2}{\sqrt{1-4x^2}}$

10



The value of x in the above diagram is:

(A) $\dfrac{3}{2}$

(B) $\dfrac{8}{3}$

(C) 6

(D) $\dfrac{2}{3}$

Section II

70 marks
Attempt Questions 11 – 14
Allow about 1 hours and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11(15 marks) Use a SEPARATE writing booklet

(a) Evaluate $\int_0^2 \frac{4}{\sqrt{4-x^2}}\,dx$ **3**

(b) Differentiate $\cos^{-1}(\sin x)$ and fully simplify. **2**

(c) Solve $x-3<\frac{4}{x}$ **3**

(d) Use the substitution $x=\sin t$ to evaluate $\int_0^{\frac{1}{2}} \sqrt{1-x^2}\,dx$ **3**

(e) The parabola's $y=x^2$ and $y=(x-2)^2$ intersect at the point P.

(i) Find the coordinates of P **1**

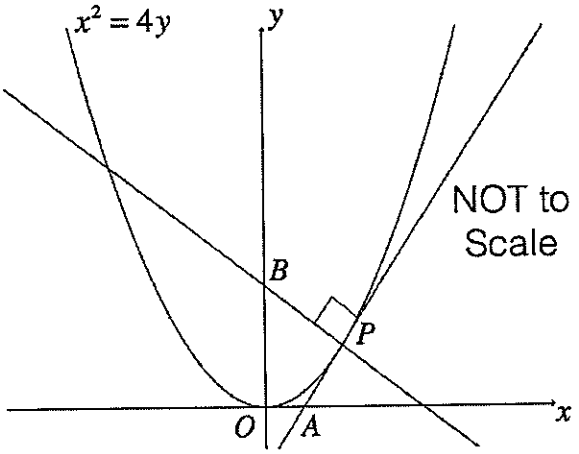
(ii) Find the angle between the tangents to the parabolas at P. Give your answer to the nearest degree. **3**

End of Question 11

Question 12(15 marks) Use a SEPARATE writing booklet

(a) Use mathematical induction to prove that $9^{n+2}-4^{n}$ is divisible by 5 for integers $n \geq 1$. **3**

(b)



The diagram shows the graph of $x^2 = 4y$. The tangent to the parabola at $P\,(2p\,,\,p^2)$, where $p > 0$ cuts the x-axis at A. The normal to the parabola at P cuts the y-axis at B.

(i) Find the equation of the tangent at P. **2**

(ii) Show that B has coordinates $\left(0\,,\,p^2+2\right)$ **2**

(iii) Let C be the midpoint of AB. Find the equation of the locus of C. **3**

(c) A particle P moving in a straight line executes Simple Harmonic Motion about a centre O. The acceleration of P is given by $\ddot{x} = -n^2 x$ where x is the distance OP and n is a constant. The amplitude of the simple harmonic motion is a.

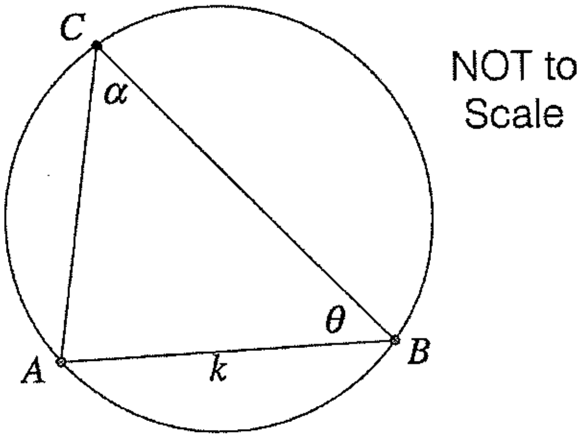
(i) Show that $v^2 = n^2\left(a^2 - x^2\right)$ **3**

(ii) Show that the motion of the particle is described by $x = a\sin\left(nt+\alpha\right)$ where α is a constant **2**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet

(a)



Points A, B, and C. lie on the circle

The length of the chord AB is a constant, k. The radian measures of $\angle ABC$ and $\angle BCA$ are θ and α respectively.

(i) Let l be the sum of the length of the chord CA and CB. Show that l is given by: **3**

$$l = \frac{k}{\sin\alpha}(\sin\theta + \sin(\theta + \alpha))$$

(ii) Explain why is α a constant? **1**

(iii) Evaluate $\frac{dl}{d\theta}$ when $\theta = \frac{\pi}{2} - \frac{\alpha}{2}$ **2**

(iv) Hence show that l is a maximum when $\theta = \frac{\pi}{2} - \frac{\alpha}{2}$ **1**

(b) Sketch $y = \frac{x^2 + x - 2}{x^2 - 1}$ clearly showing all important features. **3**

Question 13 continues on page 9

Question 13 (continued)

(c) Let T be the temperature of an object at time t and D be the temperature of the surrounding medium. Newton's Law of Cooling states that the rate of change of T is proportional to $T-D$

$$\text{i.e. } \frac{dT}{dt} = -k(T-D)$$

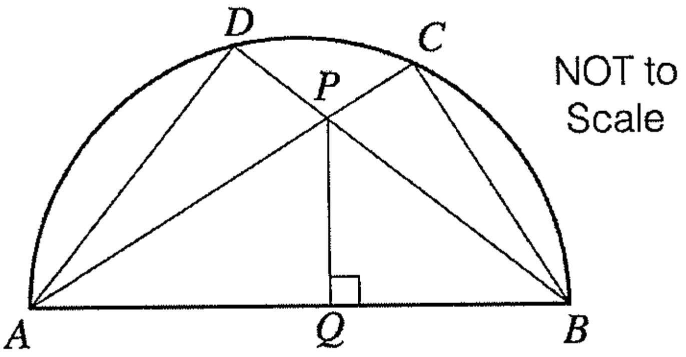
(i) Show that $T = D + Ce^{-kt}$ (where C and k are constants) satisfies Newton's Law of cooling **1**

4

(ii) A packet of meat with initial temperature of $25°\text{C}$ is placed in the freezer whose temperature is kept at a constant $-10°\text{C}$. It takes 12 minutes for the temperature of the meat to drop to $15°\text{C}$. How much *additional* time is needed for the temperature of the meet to fall to $0°\text{C}$. Give your answer in minutes correct to the nearest second.

End of Question 13

Question 14(15 marks) Use a SEPARATE writing booklet

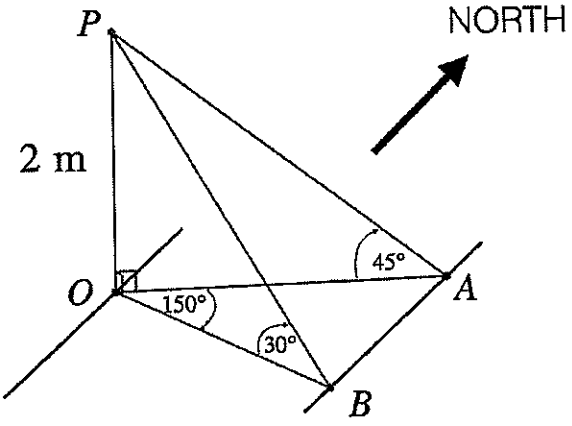


(a) AB is the diameter of a semi-circle and $PQ \perp AB$

(i) Explain why $AQPD$ and $BQPC$ are cyclic quadrilaterals **1**

(ii) Hence or otherwise prove that PQ bisects $\angle DQC$ **3**

(b) A vertical pole of height 2 m with base at point O stands on the West side of a canal that has straight parallel sides running from North to South. Two points A and B both lie on the East side of the canal, A to the north of the pole and B to the south of the pole, such that $\angle AOB = 150°$. The angles of elevation of the top of P of the pole. The angles of elevation to the top P of the pole are $45°$ from A and $30°$ from B.



Find:

(i) The exact distance AB **2**

(ii) The width of the canal (the perpendicular distance between opposite banks of the canal). Answer to the nearest centimetre. **3**

Question 14 continues on page 11

Question 14 (continued)

(c) A particle is projected from a point O on the ground with a velocity V and an inclination of θ to the horizontal. After time t the horizontal distances travelled by P are

$x = Vt\cos\theta$ and $y = Vt\sin\theta - \frac{gt^2}{2}$ respectively. DO NOT PROVE THIS.

(i) If $\tan\theta = \frac{1}{3}$ and P passes through the point $A\left(3a\ ,\frac{3a}{4}\right)$ show that $V^2 = 20ga$ **3**

(ii) At the instant of time when P is travelling in the horizontal direction (i.e. it has no vertical component to its motion), another particle Q is projected from O with a velocity U at an angle α to the horizontal. P and Q hit the ground at the same place and at the same time. **3**

Show that $U = \sqrt{\frac{145ga}{2}}$ and that $\alpha = \tan^{-1}\left(\frac{1}{12}\right)$

End of Examination ☺

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Student Number

Mathematics

Section I – Multiple Choice Answer Sheet

Use this multiple-choice answer sheet for questions 1 – 10. Detach this sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9
A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct
A ~~●~~ B ~~●~~ C ○ D ○

Start Here →

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

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STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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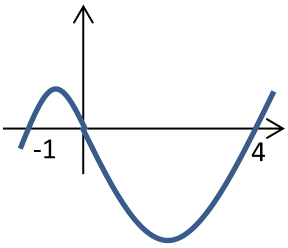
PAGE

Ex1 Mathematics Trial 2015 Solution

(1) C	(2) B	(3) C	(4) D	(5) C	(6) C	(7) B	(8) B	(9) C	(10) A

Question 11(15 marks)

(a) $\int_0^2 \frac{4}{\sqrt{4-x^2}}\,dx = 4\left[\sin^{-1}\frac{x}{2}\right]_0^2$ $= 4\left(\sin^{-1}1 - 0\right) = 4\times\frac{\pi}{2}$ $= 2\pi$	3 marks obtains a correct solution 2 marks obtains correct primitive 1 mark identifies inverse trig attempts solution
(b) $\frac{d}{dx}\cos^{\ 1}(\sin x) = \frac{-\cos x}{\sqrt{1-\sin^2 x}} = \frac{-\cos x}{\cos x} = -1$	1 correct differentiation of cos^{-1} 1 correct simplified answer
(c) $x \quad 3 < \frac{4}{x} \qquad x \neq 0$ $x(x^2 - 3x - 4) < 0$ $x(x-4)(x+1) < 0$ $\therefore \ x < -1, \quad 0 < x < 4$  	1 obtaining the cubic function 2 marks obtains one correct interval, r equivalent 3 marks correct solution
(d) $x = \sin t$ $dx = \cos t\,dt$ $\sqrt{1-x^2} = \sqrt{1-\sin^2 t}$ $= \cos t$ When $x = 0, t = 0$ $x = \frac{1}{2}, t = \frac{\pi}{6}$	1 finding all the correct substitutions including the integrand.
$\overset{\frac{1}{2}}{\underset{0}{}}\sqrt{1 \quad x^2}\,dx = \int_0^{\frac{\pi}{6}} \cos^2 t\,dt$ $= \frac{1}{2}\int_0^{\frac{\pi}{6}} \cos 2t + 1\,dt$	1 correctly integrate
$= \frac{1}{2}\left[\frac{\sin 2t}{2} + t\right]_0^{\frac{\pi}{6}}$	1 correct answer

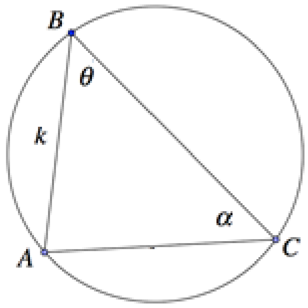
$=\frac{1}{2}\left(\frac{\sqrt{3}}{4}+\frac{\pi}{6}\right)=\frac{\sqrt{3}}{8}+\frac{\pi}{12}$	
(e) $y=x^2$ and $y=(x \quad 2)^2$ $x^2=(x-2)^2$ $4x-4=0$ $x=1, y=1$ $P\ (1, 1)$	1 correct coordinates of *P*
$\frac{d}{dx}x^2=2x$ $\qquad \frac{d}{dx}(x-2)^2=2x-4$ At $x=1$ $m_1=2$ $\qquad m_2=-2$	1 correct gradients
$\theta=\tan^{-1}\left\|\frac{2+2}{1+2\times -2}\right\|$	1 correct sub. into formula
$=53°$	1 marks answer correct to nearest degree ($\frac{0}{2}$ for using incorrect formula)

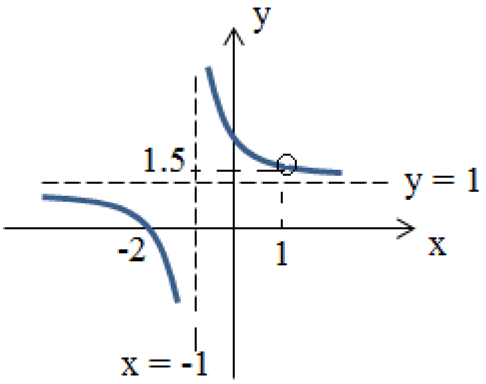
Question 12 (15 marks)

(a) $9^{n+2} \quad 4^{n}$ Prove it is true for n = 1 $9^{1+2} - 4^{1} = 9^{3} - 4$ $= 725$ is divisible by 5 ∴ It is true for n = 1	1 proving n = 1
Assume it is true for n = k ie. $9^{k+2} - 4^{k} = 5P$, where P is an integer	1 assumption and R.Q.T $9^{k+2} - 4^{k} = 5P$
R.T.P it is true for n = k+1 ie. $9^{k+3} - 4^{k+1} = 5Q$, where Q is an integer	
$9.\,9^{k+2} - 4^{k+1} = 9^{k+2}(5 + 4) - 4^{k+1}$ $= 5.9^{k+2} + 4.9^{k+2} - 4^{k+1}$ $= 5.9^{k+2} + 4(9^{k+2} - 4^{k})$ by assumption $= 5.9^{k+2} + 4(5P)$ $= 5(9^{k+2} + 4P)$ $= 5Q$ where $Q = 9^{k+2} + 4P$ ∴ Statement is true for n = k+1, ∴ Statement is true for all $n \geq 1$ by mathematical induction.	1 correct process to obtain the result.
(b) (i) $y = \frac{x^{2}}{4}$ $\frac{dy}{dx} = \frac{x}{2}$	1 finding gradient
At x = 2p, m = p Equation of the tangent is $y - p^{2} = p(x - 2p)$ $px - y - p^{2} = 0$	1 correct equation of tangent at P with full working
(ii) gradient of the normal is $-\frac{1}{p}$ equation of the normal is $y - p^{2} = -\frac{1}{p}(x - 2p)$ $py + x - p^{3} - 2p = 0$	1 correct equation of normal
when x = 0, $y = p^{2} + 2$ ∴ B has coordinates $(0, p^{2} + 2)$	1 correct coordinates of B
(iii) When y = 0, x = p ∴ A has coordinates (p, 0)	1 correct coordinates of A
The midpoint of AB is	1 correct mid-point of AB

$M\left(\frac{p+0}{2}, \frac{0+p^2+2}{2}\right)$ $x = \frac{p}{2}$ and $y = \frac{p^2+2}{2}$ p = 2x sub. into y $y = \frac{4x^2+2}{2}$ $\therefore$ $y = 2x^2 + 1$ is the locus of P.	1 correct Cartesian equation of locus of M
(c) (i) $\ddot{x} = n^2x$	1 using correct formula of $\ddot{x}$ using $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$
$\frac{d}{dx}\frac{1}{2}v^2 = -n^2x$ $\int \frac{d}{dv}\frac{1}{2}v^2 = -\int n^2x\,dx$	1 correct integration
$\frac{1}{2}v^2 = -\frac{1}{2}n^2x^2 + C$ When $v = 0, x = a$ $\therefore C = \frac{1}{2}n^2a^2$	1 correct value of C
$\therefore v^2 = n^2a^2 - n^2x^2$ $= n^2(a^2 - x^2)$ (ii) $x = a\sin(nt + \alpha)$	
From (i) $v = \pm n\sqrt{a^2 - x^2}$ $\frac{dx}{dt} = \pm n\sqrt{a^2 - x^2}$	1 signification progress towards establishing the integral
$n\frac{dt}{dx} = \frac{\pm 1}{\sqrt{a^2 - x^2}}$ $\int n\ dt = \int \frac{\pm 1}{\sqrt{a^2 - x^2}}\ dx$ $nt = \sin^{-1}\left(\frac{x}{a}\right) + C$ or $nt = \cos^{-1}\left(\frac{x}{a}\right) + C$ *We only need to consider positive value of v for the solution.* $\therefore nt + \alpha = \sin^{-1}\left(\frac{x}{a}\right)$ *where* $\alpha = -C$ $\frac{x}{a} = \sin(nt + \alpha)$ *Hence* $x = a\sin(nt + \alpha)$	1 correct integration and simplification to achieve desired result

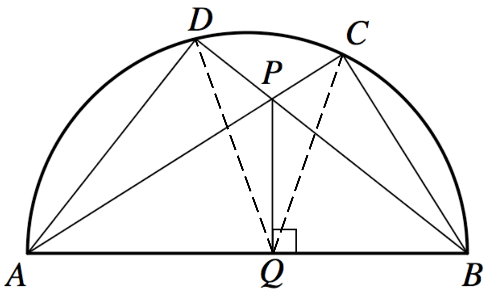
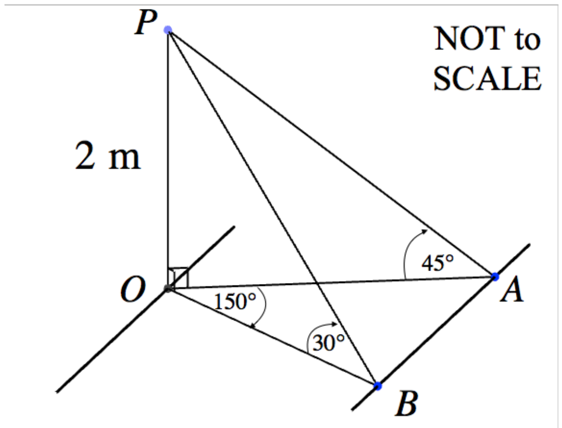
Question 13 (15 marks)

(a) (i)  $l = CA + CB$ $\frac{CA}{\sin\theta}=\frac{k}{\sin\alpha}$ $CA=\frac{k\sin\theta}{\sin\alpha}$	1 correct length of CA with working
$\frac{CB}{\sin(180°-(\alpha+\theta))}=\frac{k}{\sin\alpha}$ $CB=\frac{k\sin(\alpha+\theta)}{\sin\alpha}$	1 correct length of CB with working
$CA + CB = \frac{k\sin\theta}{\sin\alpha}+\frac{k\sin(\alpha+\theta)}{\sin\alpha}$ $\therefore\ l=\frac{k}{\sin}\left(\sin\ \ +\sin(\ \ +\ \ )\right)$	1 correct length of l
(ii) *α is the angle opposite the side AB which is a constant k. Therefore α is a constant.*	1 correct reason
(iii) $l=\frac{k}{\sin}\left(\sin\ \ +\sin(\ \ +\ \ )\right)$ $\frac{dl}{d\theta}=\frac{k}{\sin\alpha}(\cos\theta+\cos(\theta+\alpha))$	1 correct derivative of l
$Sub.\ \ =\frac{}{2}\ \ \frac{}{2}$ $\Rightarrow \frac{dl}{d\theta}=\frac{k}{\sin\alpha}\left(\cos\left(\frac{\pi}{2}-\frac{\alpha}{2}\right)+\cos\left(\frac{\pi}{2}-\frac{\alpha}{2}+\alpha\right)\right)$ $=\frac{k}{\sin\alpha}\left(\sin\frac{\alpha}{2}+\cos\left(\frac{\pi}{2}+\frac{\alpha}{2}\right)\right)$ $=\frac{k}{\sin\alpha}(\sin\frac{\alpha}{2}-\sin\frac{\alpha}{2})=0$	1 showing $\frac{dl}{d\theta}=0$
(iii)	

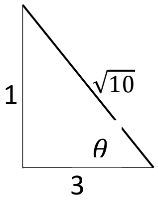
$\frac{d^2l}{d\theta^2}=\frac{k}{\sin\alpha}(-\sin\theta-\sin(\theta+\alpha)$ $=-\frac{k}{\sin\alpha}(\sin\theta+\sin(\theta+\alpha))$ <0 *Also sinθ > 0 and sin(θ + α)>0, as* *θ is acute and θ + α <* 180^0 $\frac{dl}{d\theta}=0$ when $=\frac{}{2}\quad\frac{}{2}$ (from ii) ∴ l is a maximum when $=\frac{}{2}\quad\frac{}{2}$ *(using (ii))*	1 showing $\frac{d^2l}{d\theta^2}<0$ with conclusion
(c) $y=\frac{x^2+x\quad 2}{x^2\quad 1}$ $=\frac{(x+2)(x-1)}{(x+1)(x-1)}$ $=\frac{x+2}{x+1}$ • Vertical asymptote x = -1 • Horizontal asymptote y = $\lim_{x\to\infty}\frac{x+2}{x+1}=1$ • The graph is discontinuous at x = *1*and $y=\lim_{x\to 1}\frac{x+2}{x+1}=\frac{3}{2}$ • *x-intercept is at x = -2* • *y-intercept is at y = 2* 	1 mark for correct asymptotes 1 mark for correct discontinuity 1 mark for correct shape if x and y intercepts not labelled only subtract 1 mark
(c) (i) $T=D+Ce^{\quad kt}$ $\frac{dT}{dt}=-kCe^{-kt}$ $=-k(Ce^{-kt}+D-D)$ $=-k(T-D)$ ∴ the equation satisfies Newton's law of cooling.	1 showing correct result

(ii) $T = -10 + Ae^{-kt}$ When $t = 0, T = 25 \Rightarrow 25 = -10 + A$ $\therefore\ A = 35$ When $t = 12$mins, $T = 15^o$ $15 = -10 + 35e^{-k\times 12}$ $k = \ln(25 \div 35) \div -12$ $= 0.02804$ When $T = 0$ $0 = -10 + 35e^{-0.02804t}$ $t = \ln(10 \div 35) \div -0.02804$ $= 44.677709$ mins =44mins and 41 secs *So it takes 32 minutes and 41 seconds additional time for the meat to drop to 0 degrees*	 1 correct value of A 1 correct value of k 1 correct substitution into formula 1 correct answer (additional time)

Question 14 (15 marks)

(a)  	
(i) $\angle ADB = 90^{o}$ (angle in a semi-circle) $\angle AQP = 90^{o}$ (given) $\angle ADB + \angle AQP = 180^{o}$	1 showing sum of opposite angles is 180^{o} with reason.
∴ ADPQ is a cyclic quadrilateral (sum of opposite angles is 180^{0}) Similarly QPCB is also a cyclic quadrilateral.	
(ii) $\angle DAC = \angle CBD$ (angles in same segment subtended by arc DC) (1)	1 showing $\angle DAC = \angle CBD$ with reason
$\angle DQP = \angle DAC$ (angles in same segment subtended by arc DP, using (i))	1 showing $\angle DQP = \angle DAC$ with reason
$\angle CQP = \angle CBD$ (angles in same segment subtended by arc PC, using (i))	1 showing $\angle CQP = \angle CBD$ with reason
∴ $\angle DQP = \angle CQP$ (using (1)) Hence QP bisects $\angle CQD$	-1 if no conclusion
(b)  	
(i) $OB = \dfrac{2}{\tan 30} = 2\sqrt{3}$ m $OA = \dfrac{2}{\tan 45} = 2$m	1 correct values of OA & OB
$AB^2 = (2\sqrt{3})^2 + 2^2 - 2\times 2\times 2\sqrt{3}\times 1\times \cos 150^{o}$ $= 28$ $AB = \sqrt{28}$ $= 5.29$m	1 correct value of AB

(ii) $\frac{\sin 150}{\sqrt{26}} = \frac{\sin \angle OAB}{2\sqrt{3}}$ $\sin \angle OAB = \frac{2\sqrt{3}\sin 150}{\sqrt{28}}$	1 correct use of sin rule to find sin ∠OAB
$h = 2 \times \sin \angle OAB$	1 mark finding h
$= 2 \times \left(\frac{2\sqrt{3}\sin 150}{\sqrt{28}} \right)$ $= 0.65m$ is the width of the cannal.	1 correct value of h to nearest cm
(c) (i) $x = Vt\cos$ and $y = Vt\sin \quad \frac{gt^2}{2}$ $t = \frac{x}{V\cos\theta}$ sub. into y $y = x\tan\theta - \frac{g}{2}\left(\frac{x}{V\cos\theta}\right)^2$ $= x\tan\theta - \frac{gx^2}{2V^2}\left(\tan^2\theta + 1\right) \quad (1)$	1 finding Cartesian equation of the flight path of P
Sub. $\tan \ = \frac{1}{3}$ and $A\left(3a, \frac{3a}{4}\right)$ into (1)	1 correct substitution
$\frac{3a}{4} = 3a \times \frac{1}{3} - \frac{9a^2 g}{2V^2}\left(\frac{1}{3^2} + 1\right)$ $27aV^2 = 36aV^2 - 180a^2 g$ $V^2 = \frac{180a^2 g}{9a} = 20ag$	1 showing correct result
(ii) At the instant when *P* is travelling in the horizontal direction (i.e. it has no vertical component to its motion), this happens when P reaches the highest point, i.e. when $v_y = 0$, where $v_y = V\sin\theta - gt$ The time for P to reach the highest point is: $0 = V\sin\theta - gt$ $\therefore \ t = \frac{V\sin\theta}{g}$ Sub. $\sin\theta = \frac{1}{\sqrt{10}}$ and $V = \sqrt{20ga}$	

$\Rightarrow\ t=\frac{1}{g}\sqrt{\frac{20ga}{10}}=\sqrt{\frac{2a}{g}}$ is the time for the 2nd particle to reach max. range. Hence the time for P to reach the maximum range is $t=2\sqrt{\frac{2a}{g}}$	
And the maximum range for P is  x = Vtcosθ $=\sqrt{20ga}\times2\sqrt{\frac{2a}{g}}\times\frac{3}{\sqrt{10}}$ $=12a$ The time for the second particle to reach the point (12a, 0) is t = $\sqrt{\frac{2a}{g}}$ Sub. x =12a and t = $\sqrt{\frac{2a}{g}}$ into x = Utcosα $U\cos\alpha=\frac{12a}{U}\times\sqrt{\frac{g}{2a}}$ $=\sqrt{72ga}$ (2) Sub. y = 0 and t = $\sqrt{\frac{2a}{g}}$ into y = Utsinα – $\frac{1}{2}$ gt^2 $0=U\sin\alpha\times\sqrt{\frac{2a}{g}}-\frac{g}{2}\times\frac{2a}{g}$	1 finding the max. range for 2nd particle
$\Rightarrow\ U\sin\alpha=a\times\sqrt{\frac{g}{2a}}=\sqrt{\frac{ag}{2}}$ (3) $(2)^2+(3)^2$ $\Rightarrow U^2(\cos^2\alpha+\sin^2\alpha)=72ga+\frac{ag}{2}$ $=\frac{145ga}{2}$ $\therefore U=\sqrt{\frac{145ga}{2}}$ $(2)^2\div(3)^2$	1 simplifying algebraic expressions to obtain correct answer for U
	1 simplifying algebraic expressions to obtain correct answer for α

$\frac{U\sin\alpha}{U\cos\alpha} = \sqrt{\frac{ag}{2}} \div \sqrt{72ga}$ $\Rightarrow \quad = \sqrt{\frac{ag}{144ag}}$ $\therefore \tan\alpha = \frac{1}{12}$ *hence* $\alpha = \tan^{-1}\left(\frac{1}{12}\right)$	